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Low-Order Equivalent Models of Highly Augmented Aircraft Determined from Flight Data

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This paper presents the results of a study of the feasibility of using low-order equivalent mathematical models of a highly augmented aircraft, the F-8 Digital Fly-By-Wire (DFBW), for flying qualities research. Increasingly complex models were formulated and evaluated using flight data and maximum likelihood estimation techniques. The airframe was first modeled alone. Next, equivalent derivatives were used to model the longitudinal unaugmented F-8 DFBW aircraft dynamics. The most complex model of the unaugmented aircraft incorporated a pure time shift of the pilot input, a first-order lag, and the basic longitudinal airframe model. This same model was then implemented for the F-8 DFBW aircraft in a highly augmented mode. Excellent matching of the dynamics resulted for this model, indicating that low-order equivalent models that are good representations of the highly augmented F-8 DFBW aircraft can be formulated with these methods.

Nomenclature

a_n	= normal acceleration, g
C_m	= nondimensional coefficient of rolling moment
C_N	= nondimensional coefficient of normal force
g	= acceleration due to gravity, m/s^2
K	= gain
M	= dimensional coefficient of pitching moment
N	= dimensional coefficient of normal force
q	= pitching rate, deg/s
s	= Fourier transform variable
V	= velocity, m/s
α	= angle of attack, deg
δ_{ec}	= control system pitch command, deg
δ_{ep}	= pilot pitch input, cm
δ_{es}	= elevator deflection, deg
Θ	= pitch angle, deg
τ	= first-order lag parameter, s
φ	= bank angle, deg

Subscripts

eq	= equivalent
$q,\alpha,\delta_{ep},\delta_{es}$	= derivative with respect to indicated quantity
sh	= time shifted

Introduction

At one time, low-order mathematical models described most aircraft responses well enough to permit the flying qualities of various aircraft to be compared. The ability to compare aircraft flying qualities permitted standards to be formulated (for example, military specification 8785B).[1] The flying qualities studies led, in part, to a greater understanding of the requirement for better flying qualities and contributed to the development of aircraft stability and control augmentation systems. However, low-order mathematical models cannot be applied readily to highly augmented aircraft. Researchers in stability and control and flying qualities are now looking for ways to model these highly augmented aircraft, preferably with models of fairly low order, so the handling qualities criteria developed in the past can continue to be used. New criteria are being formulated as well. For example, the new military handling qualities standard, military specification 8785C,[2] is predicated on an equivalent model of the aircraft.

As of this time, most of the research into reduced-order models has investigated the possibility of using transfer function matching techniques.[3-5] Second-order models, some of which include time delays, have been investigated the most. A very limited amount of research has also been done into dynamic response matching using maximum likelihood estimation to obtain the parameters of the model.[6] The results of these research areas raised the question of whether a low-order equivalent model of a highly augmented aircraft could be formulated using maximum likelihood estimation together with flight data. To answer this question, a study was begun using flight data from the highly augmented F-8 digital fly-by-wire (DFBW) aircraft to create a model of the aircraft's longitudinal system response. This paper reports the results obtained to date.

F-8 DFBW Aircraft

The F-8 DFBW aircraft is a standard F-8C airplane which has been modified for research in digital fly-by-wire techniques, digital control system design and implementation, and handling and flying qualities studies.[7]

The modifications essentially consist of the removal of the entire mechanical control system between the stick and rudder pedals and the actuators. The mechanical control system was replaced by a digital fly-by-wire control system using onboard digital computers.[8] Two of the longitudinal control modes are of interest to this study are 1) DIRECT, and 2), command augmentation system (CAS).

DIRECT Mode

The DIRECT mode is an unaugmented mode with no feedback loops. Input shaping and gearing are performed in the computers.

Command Augmentation System Mode

The CAS mode is a highly augmented mode with feedback loops that are driven by aircraft responses q and a_n. In addition, the stick shaping and gearing in the CAS mode are different from those in the DIRECT mode.

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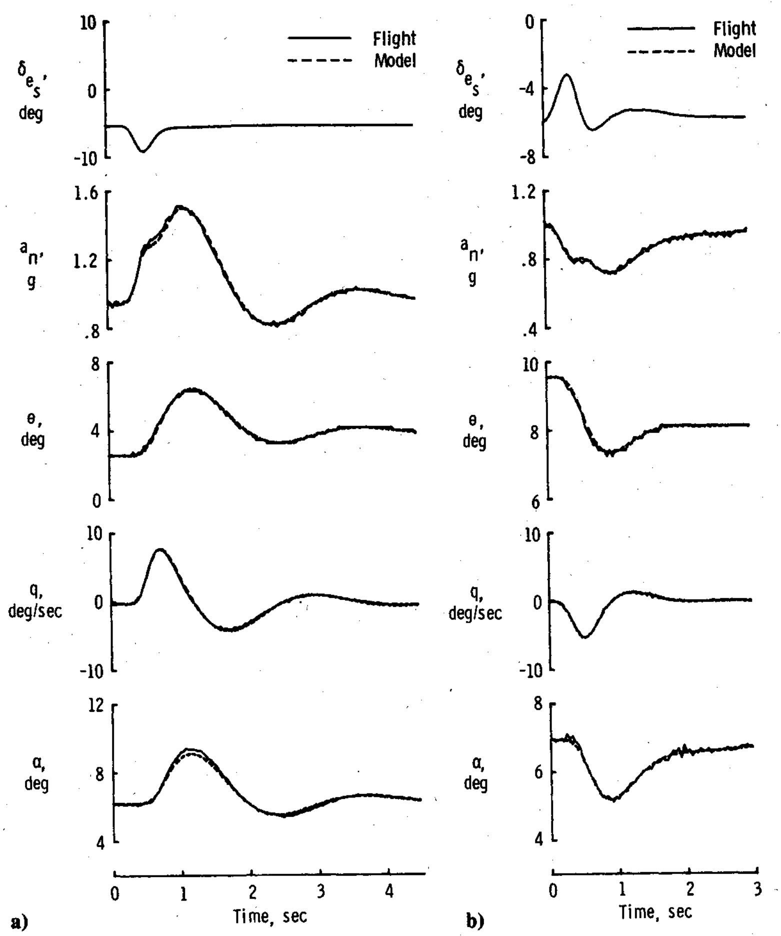

Fig. 1 Comparison of typical flight and model time histories for the F-8C airframe model; a) DIRECT maneuver and b) CAS maneuver.

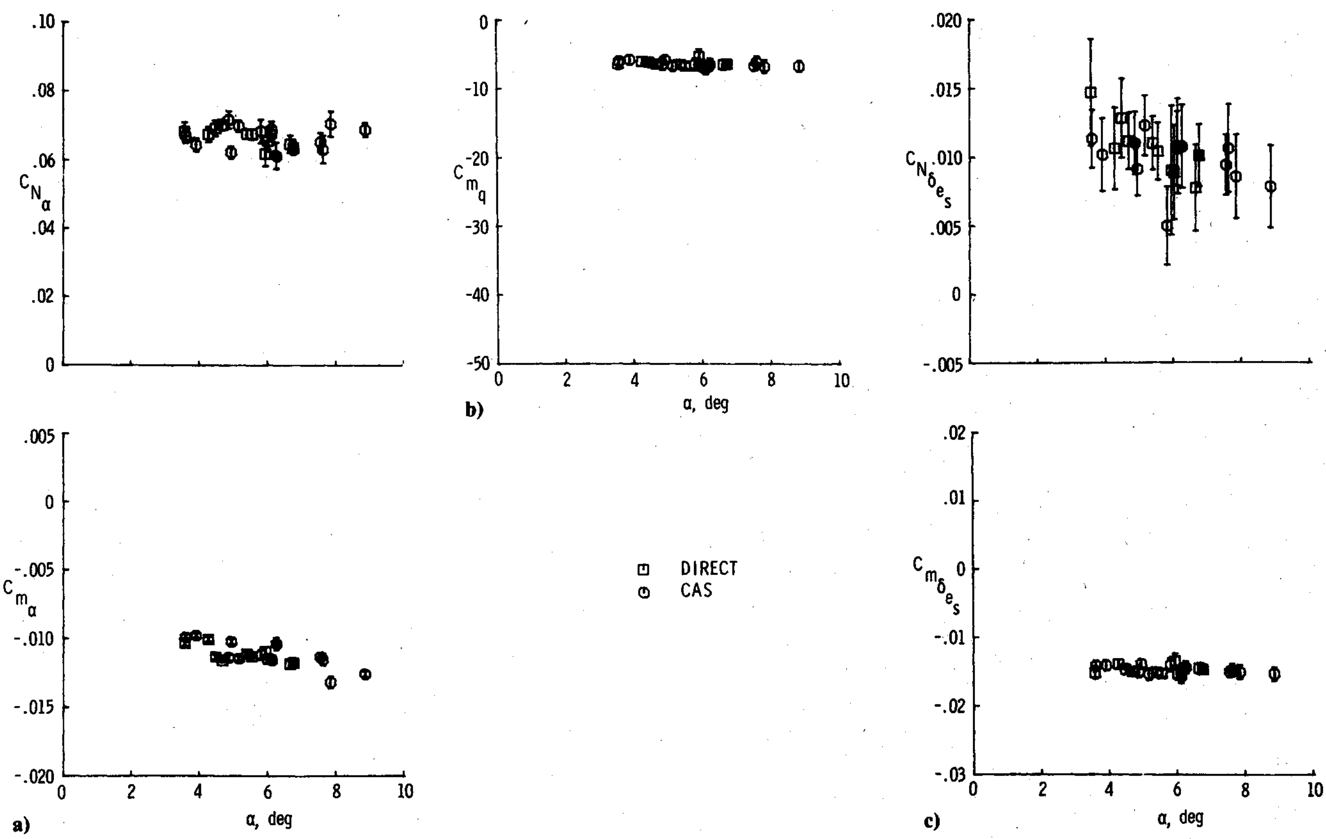

Fig. 2 Longitudinal stability and control derivatives of the F-8C airframe; solid symbols denote case shown in Fig. 1.

Because this aircraft is experimental, it is extensively instrumented and well documented. The aircraft has been used as a testbed for such experiments as a remotely augmented vehicle (RAV) study and an adaptive control systems study.[9] It has also been used for a study of the effects of control system time delays on the landing task.[10]

F-8C Airframe Model

Although the ultimate objective of these efforts is a low-order equivalent model for an entire highly augmented aircraft, this study is limited to the longitudinal axis. It was thought prudent to begin by modeling some of the individual dynamic elements of the longitudinal system, so the basic airframe was modeled separately first. The resulting model acted as a point of reference and the basis for comparison for the rest of the investigation. The airframe model is a standard mathematical model.[11]

At the Dryden Flight Research Facility, stability and control derivative estimation from flight data is done with a computer program called MMLE3.[11,12] This program is a maximum likelihood parameter estimator capable of handling bilinear dynamic equations of arbitrary order. Little difficulty was encountered in determining the airframe model. The longitudinal aircraft equations of motion used, are in state space form,

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -N_\alpha & 1 & (-\sin\theta)(\cos\phi)g/V \\ M_\alpha & M_q & 0 \\ 0 & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} -N_{\delta_{e_s}} \\ M_{\delta_{e_s}} \\ 0 \end{bmatrix} [\delta_{e_s}] \tag{1}$$

$$\begin{bmatrix} \alpha \\ q \\ \Theta \\ a_n \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ (V/g)N_\alpha & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ (V/g)N_{\delta_{e_s}} \end{bmatrix} [\delta_{e_s}] \tag{2}$$

The airframe derivatives were extracted from a total of 25 DIRECT and CAS maneuvers that were nominally flown at Mach numbers of 0.5, 0.6, and 0.67 at 3000 m. Comparisons of the flight and computed time histories for a typical DIRECT maneuver and a typical CAS maneuver flown at Mach 0.5 are shown in Fig. 1. The DIRECT maneuver was at $\alpha = 6.77$ deg and the CAS maneuver was at $\alpha = 6.30$ deg. Good agreement between the flight and computed time histories is necessary for obtaining valid stability and control parameter estimates. The complete set of nondimensional longitudinal stability and control derivatives for the airframe is shown in Fig. 2. The uncertainty levels or Cramér-Rao bounds[13] give an analytical estimate of the accuracy of the derivatives. These uncertainty levels are indicated in the plots by the vertical bars. It was possible to extract the derivatives for the basic airframe from highly augmented responses because the actual control surface position δ_{e_s} is measured. The uncertainty levels and scatter are very small for these estimates and the flight and model time histories compare quite well. Thus, these estimates were considered to be excellent. As expected, there are no significant differences between the estimates for DIRECT and CAS.

Combined System Models

After modeling the basic airframe, an attempt was made to use flight data and MMLE3 analysis to formulate and determine a low-order equivalent model for the entire airframe and control system configuration. The system may be represented as shown in Fig. 3. Note that δ_{e_s} is measured, whereas $\delta_{e_{s_{eq}}}$ is not. In the DIRECT mode there is no augmentation, and δ_{e_s} and $\delta_{e_{s_{eq}}}$ are the same.

Two approaches were taken in modeling the combined system. The first was the equivalent derivative model, where the system model has the same form as the airframe model but the input is pilot input rather than the surface position. In this model, all effects of shaping and gearing, actuator dynamics, and feedback paths are subsumed by the equivalent airframe model, and the equivalent control system is just $\delta_{e_{s_{eq}}} = \delta_{e_p}$.

In the second approach, the equivalent control system model is a time shift and a first-order lag, which together model the dynamic effect of the actuator and airborne computer. The equivalent airframe model in this form includes the effects of augmentation and time delays in the feedback path.

DIRECT Mode Models

The investigations began by using the DIRECT mode. The DIRECT mode was analyzed as a prelude to the analysis of the CAS mode model.

Equivalent Derivatives Model

The first effort at equivalent modeling was an attempt to determine whether the F-8 DFBW in the DIRECT mode could be represented by using only equivalent derivatives. That is, an attempt was made to model the entire system in the same form as the airframe model [Eqs. (1) and (2)], except that the input is δ_{e_p} rather than δ_{e_s} and the derivatives of the model are equivalent derivatives. The state space form of this model is

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -N_{\alpha_{eq}} & 1 & (-\sin\theta)(\cos\phi)g/V \\ M_{\alpha_{eq}} & M_{q_{eq}} & 0 \\ 0 & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} -N_{\delta_{e_p}} \\ M_{\delta_{e_p}} \\ 0 \end{bmatrix} [\delta_{e_p}]$$

$$\begin{bmatrix} \alpha \\ q \\ \theta \\ a_n \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ (V/g)N_{\alpha_{eq}} & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ (V/g)N_{\delta_{e_p}} \end{bmatrix} [\delta_{e_p}]$$

This model was implemented in the MMLE3 program, and 11 flight maneuvers were analyzed. These 11 maneuvers were longitudinal stability and control maneuvers flown in the DIRECT mode at all three flight conditions. Figure 4 compares the flight and computed time histories for a typical maneuver (flown at Mach 0.5) that resulted from this analysis. The results are not really satisfactory.

Equivalent Derivatives Model with Time Shift

The possibility of simply accounting for the lags and delays of the control system and actuator with a time shift of the δ_{e_p} input was considered. This model is of the same form as that used in military specification 8785C. However, the method of determining the length of its time shift is different from that of the specification.

The time shift was determined by examining the δ_{e_p} and δ_{e_s} time histories. The length of time it took for the surface to move after a pilot input ranged from 100 to 160 ms. This can be attributed to the secondary actuator time delay and the digital computer time delay. Accordingly, the smaller time shift value of 100 ms was selected so that the aircraft response would not precede the pilot input. The time shift was implemented by creating a new data file in which the proper channel was offset by the appropriate time interval.[14] The equivalent derivatives analysis was repeated using the shifted pilot input $\delta_{e_{p_{sh}}}$. As Fig. 5 shows, the results are much better than those of the previous analysis, although the response of this model was considered barely adequate. A larger time shift may further improve the response of the model. However, this possibility was not investigated.

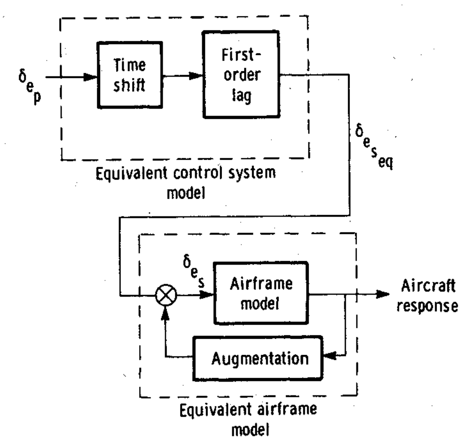

Fig. 3 Equivalent model of the aircraft.

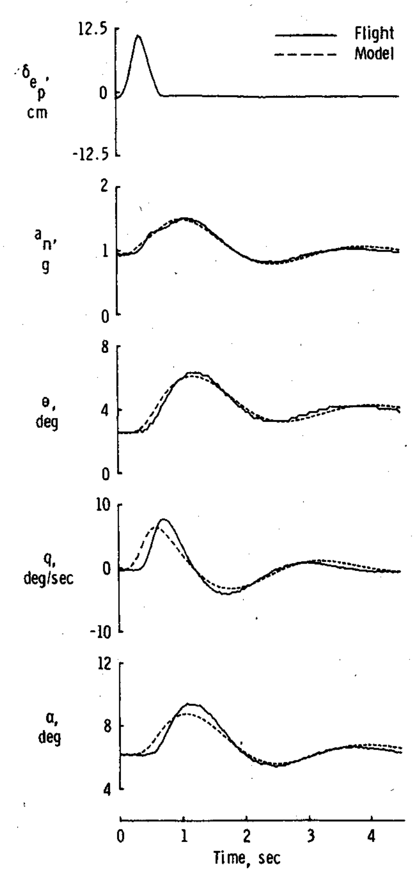

Fig. 4 Comparison of flight and model time histories using the equivalent derivatives model.

Equivalent Model with Time Shift and Lag

The next step in this study was an attempt to model the F-8 DFBW in the DIRECT mode as an equivalent control system model with a time delay and a first-order lag between δ_{e_p} and δ_{e_s} and the equivalent airframe model shown in Fig. 6. In state space form, this model is

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \\ \dot{\delta}_{e_s} \end{bmatrix} = \begin{bmatrix} -N_\alpha & 1 & (-\sin\theta)(\cos\phi)g/V & -N_{\delta_{e_s}} \\ M_\alpha & M_q & 0 & M_{\delta_{e_s}} \\ 0 & \cos\phi & 0 & 0 \\ 0 & 0 & 0 & -1/\tau \end{bmatrix} \times \begin{bmatrix} \alpha \\ q \\ \theta \\ \delta_{e_s} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ K(1/\tau) \end{bmatrix} [\delta_{e_{p_{sh}}}]$$

$$\begin{bmatrix} \alpha \\ q \\ \theta \\ a_n \\ \delta_{e_s} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ (V/g)N_\alpha & 0 & 0 & (V/g)N_{\delta_{e_s}} \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \alpha \\ q \\ \theta \\ \delta_{e_s} \end{bmatrix}$$

The value of the time shift, 100 ms, was selected in the examination of the equivalent derivatives model with time shift. The airframe derivatives in this model should be the same as those in the basic airframe model. This is because the flight and the model values of δ_{e_s} should be virtually identical. The cases used in this particular study were the 11 cases flown in the DIRECT mode, which were at all three flight conditions.

Comparisons of flight and measured time histories for this model are shown in Fig. 7 for both forward and aft stick input cases. The case in Fig. 7a was at $\alpha = 6.77$ deg, and the case in Fig. 7b was at $\alpha = 6.06$ deg. The comparison is excellent for both cases, and the model matches the dynamics of the F-8 DFBW well. However, there is an anomaly in the model responses for both cases. As shown in Fig. 7a, when the pilot input is positive (back stick pressure), the initial responses of the model δ_{e_s} lead the response of the flight q and the flight

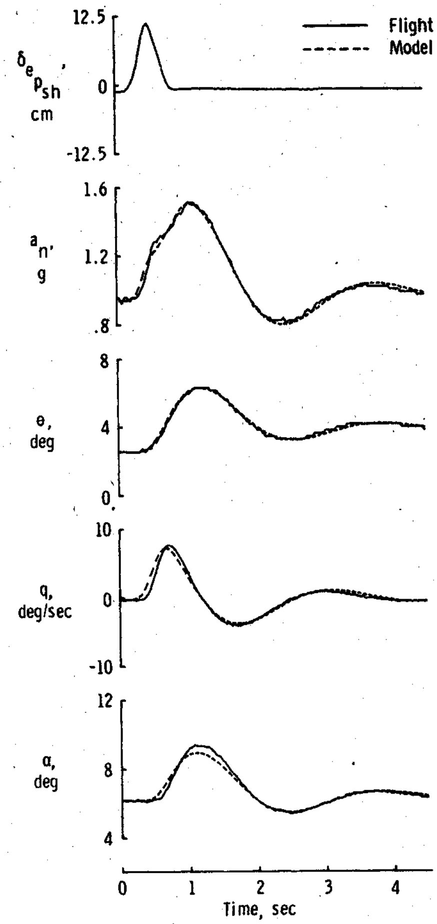

Fig. 5 Comparison of flight and model time histories using the equivalent derivatives model with time shift.

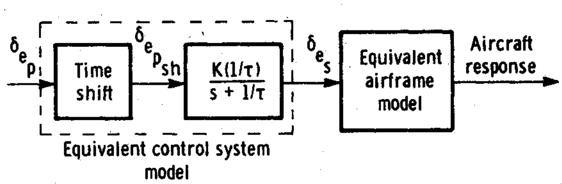

Fig. 6 Equivalent model with time shift and lag.

δ_{e_s} somewhat, although the match is excellent thereafter. This is true for all seven back stick cases. Figure 7b shows that when the pilot input is negative (forward stick pressure), the model and the flight q are essentially identical. This was true for all four forward stick cases. There is an asymmetry in the stick feel system which probably explains the anomaly.

The estimated values of the model parameters and nondimensional stability and control derivatives are shown in Fig. 8. It can be seen that the scatter and uncertainty levels for this model are somewhat larger than those for the airframe model. This is particularly noticeable for $C_{m_{\delta_{e_s}}}$. This is caused by the inevitable modeling errors introduced by use of the equivalent model. In addition, the plots of $-1/\tau$ and $K(1/\tau)$ show moderate but acceptable scatter and uncertainty levels. It is apparent from the plot of $-1/\tau$ that there are two sets of values for this parameter. The same effect appears in several other parameters, probably as a result of the feel system asymmetry.

As mentioned earlier the values of the longitudinal stability and control derivatives in this model should agree fairly well with the values of the F-8C airframe. A comparison of Figs. 2

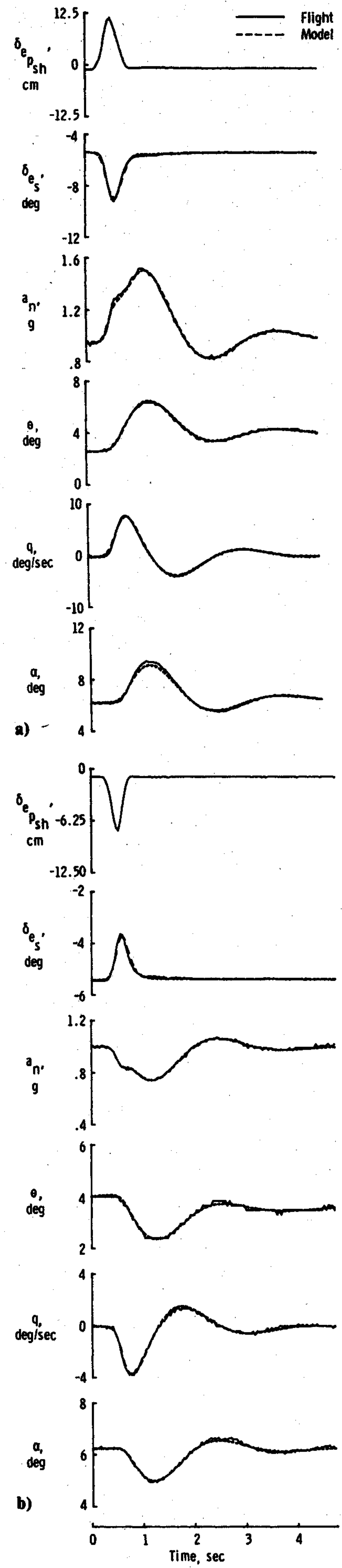

Fig. 7 Comparison of flight and model time histories using the equivalent model for DIRECT mode; a) back stick pressure, $\alpha = 6.77$ deg, b) forward stick pressure, $\alpha = 6.06$ deg.

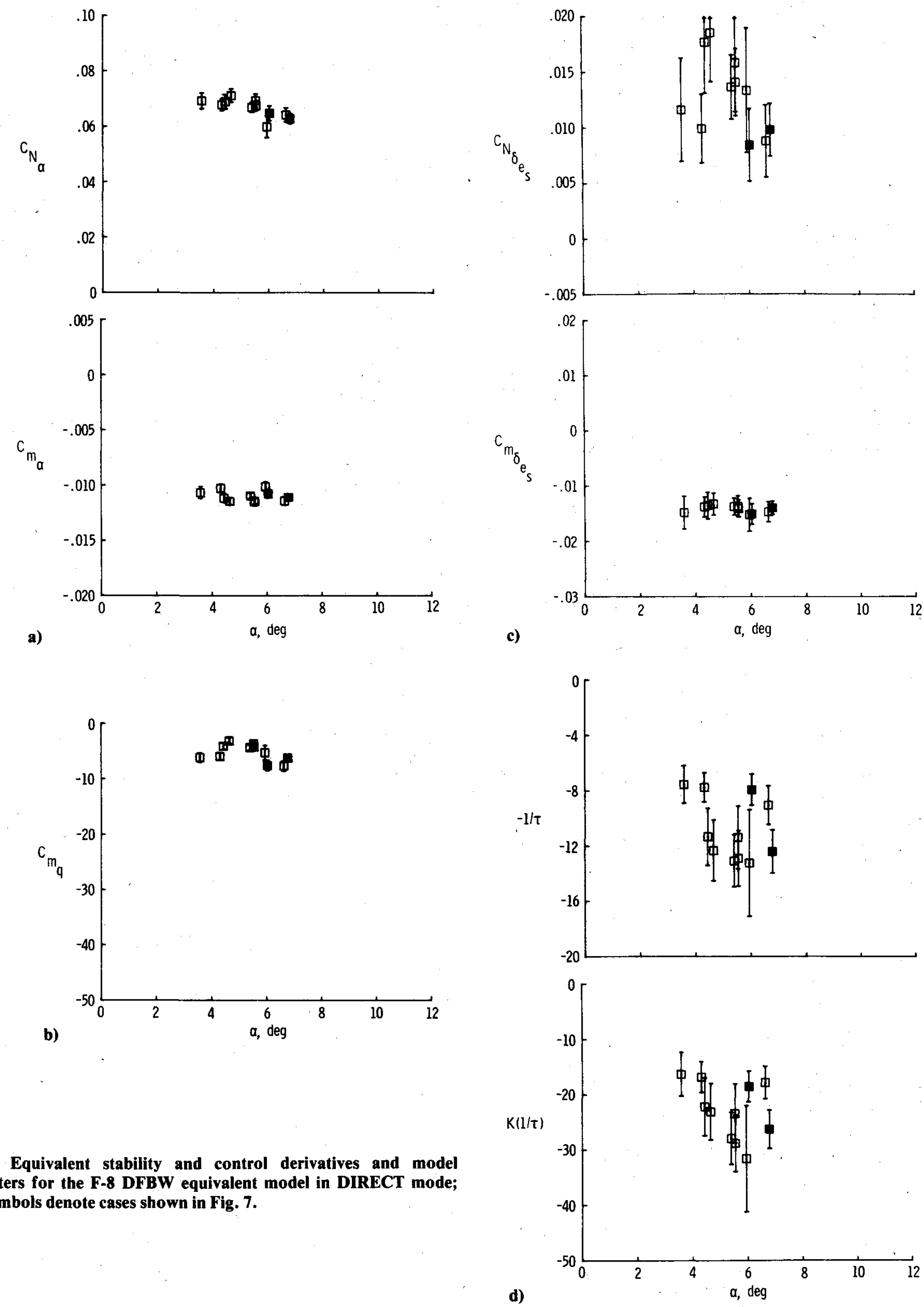

Fig. 8 Equivalent stability and control derivatives and model parameters for the F-8 DFBW equivalent model in DIRECT mode; solid symbols denote cases shown in Fig. 7.

and 8 shows that the two sets of stability and control derivatives agree fairly well, as would be expected when the dynamics of the aircraft are so well matched by the equivalent model.

CAS Mode Model

Once a good equivalent model for the DIRECT mode was determined, it seemed reasonable to assume that an equivalent model of the form shown in Fig. 6 could be developed for the CAS mode. Since the results for the DIRECT mode were so satisfactory, the model used for the CAS mode was of the same form: a time shift, a first-order lag, and the airframe model. However, the values for the time shift and the first-order lag were kept fixed at the values determined from the DIRECT mode. The derivatives in this model are equivalent derivatives. The model is of the following form.

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \\ \dot{\delta}_{e_s} \end{bmatrix} = \begin{bmatrix} -N_\alpha & 1 & (-\sin\theta)(\cos\phi)g/V & -N_{\delta_{e_{s_{eq}}}} \\ M_\alpha & M_q & 0 & M_{\delta_{e_s}} \\ 0 & \cos\phi & 0 & 0 \\ 0 & 0 & 0 & -1/\tau \end{bmatrix} \times \begin{bmatrix} \alpha \\ q \\ \theta \\ \delta_{e_{s_{eq}}} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ K(1/\tau) \end{bmatrix} [\delta_{e_{p_{sh}}}]$$

$$\begin{bmatrix} \alpha \\ q \\ \theta \\ a_n \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ (V/g)N_{\alpha_{eq}} & 0 & 0 & (V/g)N_{\delta_{e_{s_{eq}}}} \end{bmatrix} \begin{bmatrix} \alpha \\ q \\ \theta \\ \delta_{e_{s_{eq}}} \end{bmatrix}$$

where $-1/\tau = -10.0$ and $K(1/\tau) = 22.0$. Note that, unlike the DIRECT model, no δ_{e_s} observation is available.

A comparison of the flight and computed time histories is shown in Fig. 9. This maneuver was flown at Mach 0.5 with

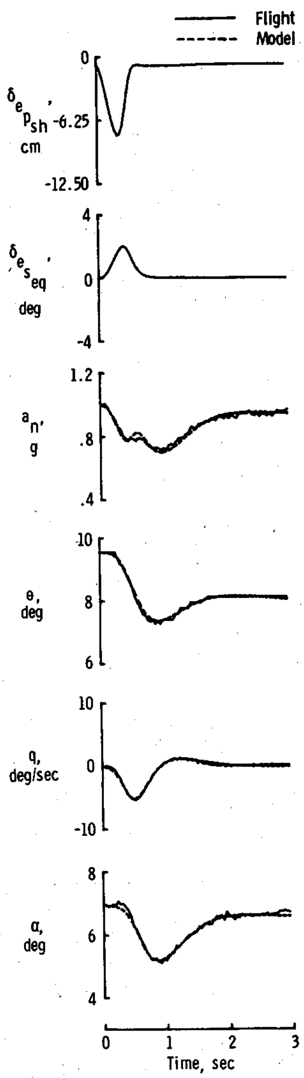

Fig. 9 Comparison of flight and model time histories using the equivalent model for CAS mode.

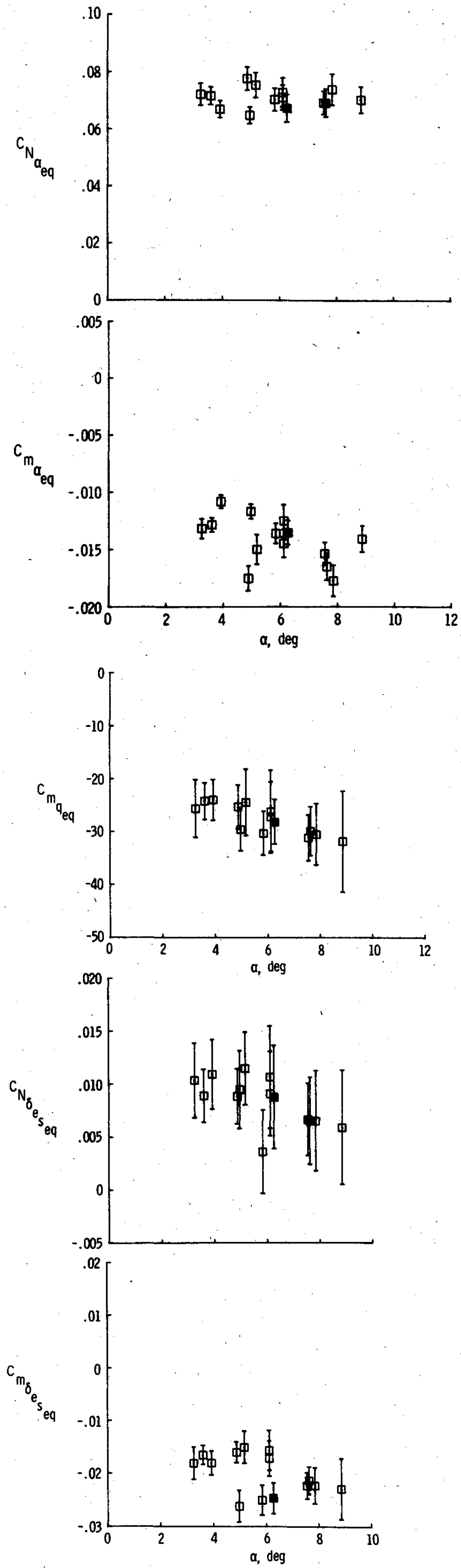

Fig. 10 Equivalent stability and control derivatives for the F-8 DFBW equivalent airframe model in CAS mode; solid symbols denote case shown in Fig. 9.

$\alpha = 6.31$ deg. This comparison shows that the dynamics of the F-8 DFBW are extremely well matched. The equivalent nondimensional longitudinal stability and control derivatives of this model are shown in Fig. 10. A comparison of the stability and control derivatives for the F-8C airframe in Fig. 2 with the equivalent derivatives of the CAS equivalent model in Fig. 10 shows differences between the values. The most striking difference is in C_{m_q}, which is about -7 for the airframe and about -28 for the CAS model. This was, of course, expected. The uncertainty levels and scatter are larger than those of the DIRECT model. This scatter results from the use of a very simple model to represent the complicated CAS. The extremely good match of the system dynamics, however, indicates that this model is a valid low-order equivalent model of the F-8 DFBW aircraft in the CAS mode.

Concluding Remarks

This study of the feasibility of using low-order equivalent models of the highly augmented F-8 digital fly-by-wire (DFBW) aircraft in the longitudinal axis has yielded encouraging results. The first area considered was the modeling of the airframe. The results for this modeling were very good, but this did not address the complete problem, as it is not usable for augmented vehicles. The next approach was the use of equivalent derivatives with and without a time shift. The results were adequate, but it was felt that better results could be obtained using a slightly more complex model. This model, incorporating a pure time shift, a first-order lag, and the basic longitudinal airframe model, was first implemented for the F-8 DFBW in the DIRECT mode. The results were good, in that the dynamics were excellently matched. Using the values for the time shift and the first-order lag determined in the DIRECT mode, the same model was implemented for the command augmentation system (CAS) mode. Again, the match of the dynamics obtained was excellent. These results are quite encouraging, and further investigation of this low-order equivalent model are in progress.

This study preceded the promulgation of military specification 8785C and, although the equivalent derivatives model with time shift is the same as the model in the specification, the means of determining the length of the time shift is not. An investigation into estimation of the time shift, as well as the equivalent derivatives, is in progress.

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